AQA-Core 2

Chapter-wise Past Exam Questions

Index

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Algebra- INDICES

Q1. (a) Write $\sqrt[4]{x^3}$ in the form x^k. **(1)**

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$. **(2)**

(Total 3 marks)

Q2. (a) Write down the values of p, q and r given that:

(i) $8 = 2^p$; **(1)**

(ii) $\dfrac{1}{8} = 2^q$; **(1)**

(iii) $\sqrt{2} = 2^r$. **(1)**

(b) Find the value of x for which. $\sqrt{2} \times 2^x = \dfrac{1}{8}$ **(2)**

(Total 5 marks)

Q3. (a) Write down the values of *p*, *q* and *r* given that:

(i) $64 = 8^p$;

(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. **(3)**

(b) Find the value of *x* for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$

(2)
(Total 5 marks)

Further Differentiation

Q1. A curve has the equation

$$y = \frac{12 + x^2\sqrt{x}}{x}, \quad x > 0$$

(a) Express $\frac{12 + x^2\sqrt{x}}{x}$ in the form $12x^p + x^q$. **(3)**

(b) (i) Hence find $\frac{dy}{dx}$. **(2)**

(ii) Find an equation of the normal to the curve at the point on the curve where $x = 4$. **(4)**

(iii) The curve has a stationary point P. Show that the x-coordinate of P can be written in the form 2^k, where k is a rational number. **(3)**

(Total 12 marks)

Q2. A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, x > 0$$

(a) Express $\frac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$. **(3)**

(b) (i) Hence find $\frac{dy}{dx}$. **(2)**

(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$. **(4)**

(c) (i) Find $\frac{d^2y}{dx^2}$. **(2)**

(ii) Hence deduce that the curve C has no maximum points. **(2)**

(Total 13 marks)

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Q3. A curve is defined, for $x > 0$, by the equation $y = f(x)$, where

$$f(x) = \frac{x^8 - 1}{x^3}$$

(a) Express $\frac{x^8 - 1}{x^3}$ in the form $x^p - x^q$, where p and q are integers. **(2)**

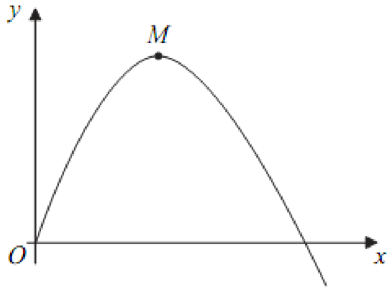
(b) (i) Hence differentiate $f(x)$ to find $f'(x)$. **(2)**

(ii) Hence show that f is an increasing function. **(2)**

(c) Find the gradient of the normal to the curve at the point (1, 0). **(3)**

(Total 9 marks)

Q4. The diagram shows part of a curve with a maximum point *M*.



The curve is defined for $x \geq 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\frac{dy}{dx}$. **(3)**

(b) (i) Hence find the coordinates of the maximum point *M*. **(3)**

(ii) Write down the equation of the normal to the curve at *M*. **(1)**

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point *P*, giving your answer in the form $ax + by = c$, where *a*, *b* and *c* are positive integers. **(4)**

(ii) The normal to the curve at the points *M* and *P* intersect at the point *R*. Find the coordinates of *R*. **(2)**

(Total 13 marks)

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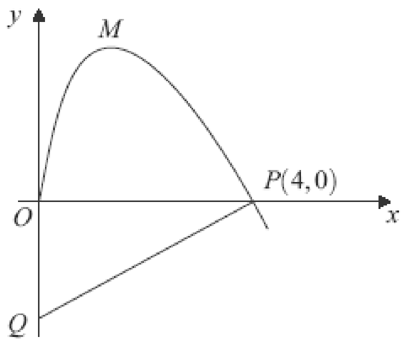
Q5. A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.
The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geq 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(i) Find $\dfrac{dy}{dx}$. **(3)**

(ii) Show that the gradient of the curve at $P(4, 0)$ is –2. **(2)**

(iii) Find an equation of the normal to the curve at $P(4, 0)$. **(3)**

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. **(3)**

(v) The curve has a maximum point M. Find the x-coordinate of M. **(3)**

(Total 14 marks)

ANSWERS

MARK SCHEME – Indices

M1.(a) $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

Accept k = ¾ OE

B1 **1**

(b) $\frac{1-x^2}{\sqrt[4]{x^3}} = \frac{1}{\sqrt[4]{x^3}} - \frac{x^2}{\sqrt[4]{x^3}} = x^{-k} - \frac{x^2}{\sqrt[4]{x^3}}$ [or $\frac{1}{\sqrt[4]{x^3}} - x^{2-k}$]

Split followed by at least one correct index law used to remove denominator.

M1

$= x^{-\frac{3}{4}} - x^{\frac{5}{4}}$

If incorrect, ft on c's non-integer k value answer to part (a), provided M1 has been awarded. Accept answer given in form of values for p and q.

A1F **2** **[3]**

M2. (a) (i) (*p* =) 3

B1 **1**

(ii) (*q* =) –3

If not correct, ft on −p

B1F **1**

(iii) $(r =)\frac{1}{2}$

OE

B1 **1**

(b) $2^{\frac{1}{2}} \times 2^x = 2^{-3} \Rightarrow 2^{\frac{1}{2}+x} = 2^{-3}$

Using a law of indices or logs correctly to combine at least two of the powers of 2 PI

M1

$\Rightarrow x = -3\frac{1}{2}$

If not correct, ft on x = q − r provided method shown

A1F

2

[5]

M3. (a) (i) {p =} 2

Condone '64 = 8^2'

B1

(ii) {q =} – 2

Ft on '– p' if q not correct

B1ft

(iii) {r =} 0.5

Condone '√8 = $8^{0.5}$'

B1

3

(b) $\frac{8^x}{8^{0.5}} = 8^{-2} \Rightarrow 8^{x-0.5} = 8^{-2}$ OE

*Using parts (a) **&** valid index law to stage $8^c = 8^d$ (PI)*

M1

$\Rightarrow x - 0.5 = -2 \quad \Rightarrow x = -1.5$

Ft on c's (q + r) if not correct
(Accept correct answer without working)

A1ft

2

ALT: $\log 8^x = \log k$, $x\log 8 = \log k$; $x = -1.5$

(M1 A1)

[5]

MARK SCHEME – Further Differentiation

M1.(a) $\sqrt{x} = x^{0.5}$

$\sqrt{x} = x^{0.5}$ *or* $\sqrt{x} = x^{\frac{1}{2}}$ *seen or used*

B1

$\frac{12 + x^2\sqrt{x}}{x} = \frac{12 + x^{2.5}}{x}$

$= 12x^{-1} + x^{1.5}$

$12x^{-1}$ or $p = -1$ B1

$x^{1.5}$ or $q = \frac{3}{2}$ (=1.5)

B1 **3**

(b) (i) $\frac{dy}{dx} = -12x^{-2}$

Ft on c's p only if c's p is a negative integer

B1F

$+ 1.5x^{0.5}$

Ft on c's q only if c's q is a pos non-integer

B1F **2**

(ii) When $x = 4$, $y = 11$

B1

When $x = 4$, $\frac{dy}{dx} = \frac{-12}{16} + 3 = \frac{9}{4}$

Attempt to find $\frac{dy}{dx}$ when $x = 4$ PI

M1

Gradient of normal = $-\frac{4}{9}$

$m \times m' = -1$ used

m1

Eqn of normal: $y - 11 = -\frac{4}{9}(x - 4)$

ACF eg $4x + 9y = 115$

A1 **4**

(iii) At St Pt $\frac{dy}{dx} = -12x^{-2} + 1.5x^{0.5} = 0$

Equating c's $\frac{dy}{dx}$ to zero

M1

$\Rightarrow x^2 x^{0.5} = 8$, $\Rightarrow x^{\frac{5}{2}} = 8 \Rightarrow x = 8^{\frac{2}{5}}$

A correct eqn in the form $x^n = c$ or $x = c^{\frac{1}{n}}$ correctly obtained.

A1

$\Rightarrow x = (2^3)^{\frac{2}{5}} \Rightarrow x = 2^{\frac{6}{5}}$

CSO $x = 2^{\frac{6}{5}}$. All working must be correct and in an exact form.
If 'x = 0' also appears then A0 CSO

A1 **3** **[12]**

M2. (a) $\sqrt{x} = x^{\frac{1}{2}}$

PI

B1

$$\frac{x^3 + \sqrt{x}}{x} = \frac{x^3}{x} + \frac{\sqrt{x}}{x} = x^2 + x^{-\frac{1}{2}}$$

Accept p = 2; q = $-\frac{1}{2}$

B1;B1 **3**

(b) (i) $\frac{dy}{dx} = 2x - \frac{1}{2}x^{-\frac{3}{2}}$

Reduces both powers by 1

M1

ACF

A1 **2**

(ii) When $x = 1$, $y = 2$

PI if not stated explicitly eg the '2'
may appear in the correct posn. in later eqn.

B1

When $x = 1$, $\frac{dy}{dx} = 2 - \frac{1}{2} = \frac{3}{2}$

Attempt to find $\frac{dy}{dx}$ When x = 1 PI

M1

Gradient of normal = $-\frac{2}{3}$

–1/ (c's value of dy/dx when x = 1) either stated as the gradient of the normal or used as the gradient in the equation of the normal

m1

Equation of normal: $y - 2 = -\frac{2}{3}(x - 1)$

Only ft on c's $\frac{dy}{dx}$ *in part (b)(i)*
ACftF

A1F

4

(c) (i) $\frac{d^2y}{dx^2} = 2 + \frac{3}{4}x^{-\frac{5}{2}}$

Reduces both powers by 1.

M1

Ft on (b)(i) provided at least one power to be differentiated is both negative and fractional

A1F

2

(ii) (Since $x > 0$,) $\frac{d^2y}{dx^2} > 0$

For a maximum point $\frac{d^2y}{dx^2}$ is **not** positive so *C* has no maximum points

E1 for attempt to find the sign of $\frac{d^2y}{dx^2}$ *;*
either in general terms or at the pt(s) where c's dy/dx = 0 or the remaining E mark

a correct justification for why $\frac{d^2y}{dx^2} > 0$ *and also a full correct concluding statement must be made.*

E2,1,0

2

[13]

M3. (a) .... $= x^5 - x^{-3}$

One power correct

M1

Accept p = 5, q = –3

A1

2

(b) (i) $f'(x) = 5x^4$

ft on px^{p-1}

B1ft

$+3x^{-4}$

ft on $-qx^{q-1}$ provided q < 0

B1ft

2

(ii) $f'(x)\left\{= 5x^4 + \dfrac{3}{x^4}\right\} > 0$

M1 Considers sign of f'(x); a statement "f'(x) > 0 OE" with 'f increasing'. M1

$\Rightarrow$ f is increasing {function}

A1 needs f'(x) of the form $ax^4 + \dfrac{b}{x^4}$, where a and b both > 0 and no incorrect statements based on f'(x) at different values of x

A1

2

(c) At (1, 0), f'(1) = 5 + 3 = 8

Attempts to find f'(1)

M1

$\Rightarrow$ grad. of normal $= -\dfrac{1}{8}$

Use of m × m' = –1 PI

m1

ft on wrong f'(x)

A1ft

3

[9]

M4. (a) *For either 6 or $6x^0$* B1

$$\frac{dy}{dx} = 6 - 3x^{\frac{1}{2}}$$

$Ax^{\frac{3}{2}-1}$, A ≠ 0 OE M1

$6 - 3x^{\frac{1}{2}}$ or $6 - 3\sqrt{x}$ with no '+c'

[If unsimplified here, A1 can be awarded retrospectively if correct simplified expression is seen explicitly in (b)(i).] A1 3

(b) (i) $6 - 3x^{\frac{1}{2}} = 0$

Equating c's $\frac{dy}{dx}$ to 0 PI by correct ft

rearrangement of c's dy/dx = 0 M1

$x^{\frac{1}{2}} = 2 \Rightarrow x = 2^2$

$x^{\frac{1}{2}} = k$ (k > 0), to $x = k^2$.
PI by correct value of x if no error seen m1

$M(4, 8)$

SC If M0 award B1 for (4, 8) A1 3

(ii) Eqn of normal at M is $x = 4$

Ft on x = c's x_M B1F 1

(c) (i) When $x = \frac{9}{4}, \frac{dy}{dx} = 6 - 3 \times \frac{3}{2} = \frac{3}{2}$

Attempt to find $\frac{dy}{dx}$ when $x = \frac{9}{4}$ M1

Gradient of normal at P = $-\frac{2}{3}$

m × m′ = − 1 used m1

Eqn of normal: $y - \frac{27}{4} = -\frac{2}{3}\left(x - \frac{9}{4}\right)$

ACF eg $y = -\frac{2}{3}x + \frac{33}{4}$ A1

$12y - 81 = -8x + 18 \Rightarrow 8x + 12y = 99$

Coeffs and constant must now be positive integers, but accept different order

eg 12y + 8x = 99 A1 4

(ii) 8(4) + 12*y* = 99

Solving c's answer (b)(ii), (must be in form x = positive const), with c's answer (c)(i). PI by correct earlier work and correct coordinates for R. M1

$R\left(4, \frac{67}{12}\right)$

Accept 5.58 or better as equivalent to $\frac{67}{12}$ A1 2 **[13]**

M5. (i) $\frac{dy}{dx} = 4 \times \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{\frac{1}{2}} = 2x^{-\frac{1}{2}} - \frac{3}{2} x^{\frac{1}{2}}$

A power decreased by 1 M1

A1For each correct term A1 A1 3

(ii) At *P*(4, 0), $\frac{dy}{dx} = \frac{2}{\sqrt{4}} - \frac{3}{2} \times 2$

Attempts $\frac{dy}{dx}$ when x = 4 M1

= 1 – 3 = – 2

AG A1

2

(iii) Gradient of normal = $\frac{1}{2}$

Use of or stating m × m′ = –1 M1

Equation of normal is *y* – 0 = *m* [*x* – 4]

m numerical; can be awarded even if m = –2 M1

$y - 0 = \frac{1}{2}(x - 4) \Rightarrow \quad 2y = x - 4$

ACF of the equation	A1	3	
(iv) At Q, $x = 0$, $2y = 0 - 4$			
PI	M1		
$y_Q = -2$			
Area of triangle $OPQ = 0.5 \times 4 \times \lvert y_Q \rvert$			
Ft on a linear equation for normal provided y_Q is negative and prev A1 is lost	A1F		
= 4			
Ft on c's negative y_Q	B1F	3	
(v) $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0 \Rightarrow 2x^{-\frac{1}{2}} = \frac{3}{2}x^{\frac{1}{2}}$			
Puts c's $\frac{dy}{dx} = 0$ and a 1st step in attempt to solve.	M1		
$2 = \frac{3}{2}x, \quad \Rightarrow x = \frac{4}{3}$			
Valid method to ax = b	m1		
Condone 1.3 or better	A1	3	**[14]**